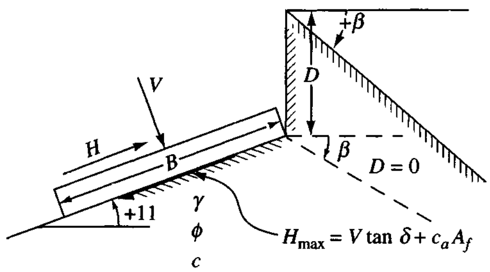
Notes: $\beta + \eta$ 90° (Both β and η have signs (+) shown.)

β ϕ



For: $L/B \le 2$ use ϕ_{tr}

$L/B > 2$ use $\phi_{ps} = 1.5\phi_{tr} - 17°$

$\phi_{tr} \le 34°$ use $\phi_{tr} = \phi_{ps}$

δ = friction angle between base and soil ($.5\phi \le \delta \le \phi$)

$A_f = B'L'$ (effective area)

c_a = base adhesion (0.6 to 1.0c)

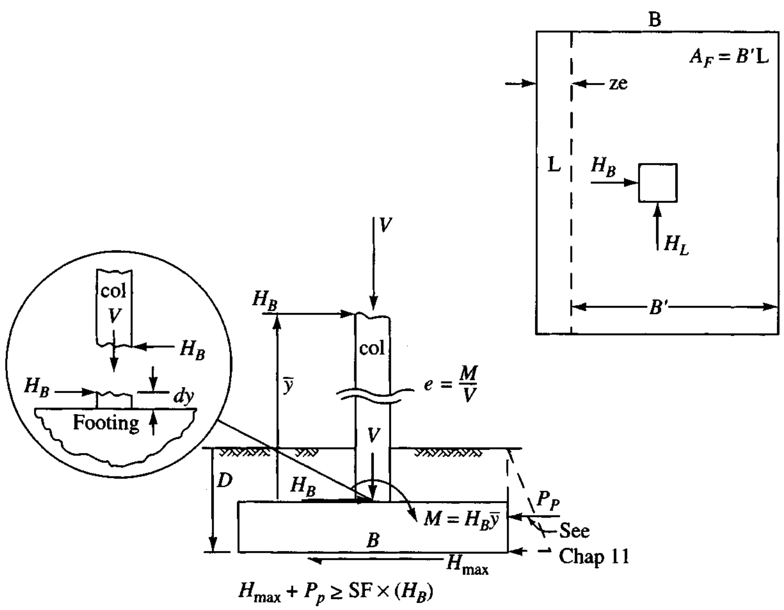


TABLE 4-5c

Table of inclination, ground, and base factors for the Vesić (1973, 1975b) bearing-capacity equations. See notes below and refer to sketch for identification of terms.

Inclination factors	Ground factors (base on slope)
$i'_c = 1 - \dfrac{mH_i}{A_f c_a N_c}$ $(\phi = 0)$	$g'_c = \dfrac{\beta}{5.14}$ β in radians
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1}$ $(\phi > 0)$	$g_c = i_q - \dfrac{1 - i_q}{5.14 \tan\phi}$ $\phi > 0$
i_q, and m defined below	i_q defined with i_c
$i_q = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^m$	$g_q = g_\gamma = (1.0 - \tan\beta)^2$
	Base factors (tilted base)
$i_\gamma = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^{m+1}$	$b'_c = g'_c$ $(\phi = 0)$
$m = m_B = \dfrac{2 + B/L}{1 + B/L}$	$b_c = 1 - \dfrac{2\beta}{5.14 \tan\phi}$
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$	$b_q = b_\gamma = (1.0 - \eta \tan\phi)^2$

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2\sin(\pm\beta)$ in N_γ term.
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use of B and L, not B', L'.
3. Refer to Table sketch and Tables 4-5a,b for term identification.
4. Terms N_c, N_q, and N_γ are identified in Table 4-1.
5. Vesić always uses the bearing-capacity equation given in Table 4-1 (uses B' in the N_γ term even when $H_i = H_L$).
6. H_i term $\le$ 1.0 for computing i_q, i_γ (always).

TABLE 4-5*a*

Shape and depth factors for use in either the Hansen (1970) or Vesić (1973, 1975*b*) bearing-capacity equations of Table 4-1. Use s'_c, d'_c when $\phi = 0$ only for Hansen equations. Subscripts H, V for Hansen, Vesić, respectively.

Shape factors	Depth factors
$s'_{c(H)} = 0.2\frac{B'}{L'} \quad (\phi = 0°)$ $s_{c(H)} = 1.0 + \frac{N_q}{N_c} \cdot \frac{B'}{L'}$ $s_{c(V)} = 1.0 + \frac{N_q}{N_c} \cdot \frac{B}{L}$ $s_c = 1.0$ for strip	$d'_c = 0.4k \quad (\phi = 0°)$ $d_c = 1.0 + 0.4k$ $k = D/B$ for $D/B \leq 1$ $k = \tan^{-1}(D/B)$ for $D/B > 1$ k in radians
$s_{q(H)} = 1.0 + \frac{B'}{L'}\sin\phi$ $s_{q(V)} = 1.0 + \frac{B}{L}\tan\phi$ for all ϕ	$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$ k defined above
$s_{\gamma(H)} = 1.0 - 0.4\frac{B'}{L'} \geq 0.6$ $s_{\gamma(V)} = 1.0 - 0.4\frac{B}{L} \geq 0.6$	$d_\gamma = 1.00$ for all ϕ

Notes:

1. Note use of "effective" base dimensions B', L' by Hansen but not by Vesić.
2. The values above are consistent with either a vertical load or a vertical load accompanied by a horizontal load H_B.
3. With a vertical load and a load H_L (and either $H_B = 0$ or $H_B > 0$) you may have to compute two sets of shape s_i and d_i as $s_{i,B}$, $s_{i,L}$ and $d_{i,B}$, $d_{i,L}$. For i, L subscripts of Eq. (4-2), presented in Sec. 4-6, use ratio L'/B' or D/L'.

TABLE 4-5*b*

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5*c* for equivalent Vesić equations.

Inclination factors	Ground factors (base on slope)
$i'_c = 0.5 - \sqrt{1 - \frac{H_i}{A_f C_a}}$	$g'_c = \frac{\beta°}{147°}$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \frac{\beta°}{147°}$
$i_q = \left[1 - \frac{0.5 H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_1}$ $2 \leq \alpha_1 \leq 5$	$g_q = g_\gamma = (1 - 0.5\tan\beta)^5$
	Base factors (tilted base)
$i_\gamma = \left[1 - \frac{0.7 H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$	$b'_c = \frac{\eta°}{147°} \quad (\phi = 0)$
$i_\gamma = \left[1 - \frac{(0.7 - \eta°/450°)H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$ $2 \leq \alpha_2 \leq 5$	$b_c = 1 - \frac{\eta°}{147°} \quad (\phi > 0)$ $b_q = \exp(-2\eta\tan\phi)$ $b_\gamma = \exp(-2.7\eta\tan\phi)$ η in radians

Notes:

1. Use H_i as either H_B or H_L, or both if $H_L > 0$.
2. Hansen (1970) did not give an i_c for $\phi > 0$. The value above is from Hansen (1961) and also used by Vesić.
3. Variable c_a = base adhesion, on the order of 0.6 to 1.0 × base cohesion.
4. Refer to sketch for identification of angles η and β, footing depth D, location of H_i (parallel and at top of base slab; usually also produces eccentricity). Especially note V = force *normal* to base and is not the resultant R from combining V and H_i.

$$\left.\begin{aligned} d_c &= 1 + 0.4\tan^{-1}\frac{D}{B} \\ d_q &= 1 + 2\tan\phi(1-\sin\phi)^2\tan^{-1}\frac{D}{B} \end{aligned}\right\} \frac{D}{B} > 1$$

These expressions give a discontinuity at $D/B = 1$; however, note the use of ≤ and >. For $\phi = 0$ (giving d_c') we have

$D/B =$	0	1	1.5*	2	5	10	20	100
$d_c' =$	0	0.40	0.42	0.44	0.55	0.59	0.61	0.62

*Actually computes 0.39

We can see that use of $\tan^{-1} D/B$ for $D/B > 1$ controls the increase in d_c and d_q that are in line with observations that q_{ult} appears to approach a limiting value at some depth ratio D/B, where this value of D is often termed the critical depth. This limitation on q_{ult} will be further considered in Chap. 16 on piles.

Vesić's Bearing-Capacity Equations

The Vesić (1973, 1975*b*) procedure is essentially the same as the method of Hansen (1961) with select changes. The N_c and N_q terms are those of Hansen but N_γ is slightly different (see Table 4-4). There are also differences in the i_i, b_i, and g_i terms as in Table 4-5*c*. The Vesić equation is somewhat easier to use than Hansen's because Hansen uses the i terms in computing shape factors s_i whereas Vesić does not (refer to Examples 4-6 and 4-7 following).

Which Equations to Use

There are few full-scale footing tests reported in the literature (where one usually goes to find substantiating data). The reason is that, as previously noted, they are very expensive to do and the cost is difficult to justify except as pure research (using a government grant) or for a precise determination for an important project—usually on the basis of settlement control. Few clients are willing to underwrite the costs of a full-scale footing load test when the bearing capacity can be obtained—often using empirical SPT or CPT data directly—to a sufficient precision for most projects.

Table 4-6 is a summary of eight load tests where the footings are somewhat larger than models and the soil data are determined as accurately as possible. The soil parameters and q_{ult} (in kg/cm^2) are from Milović (1965). The several methods used in this text [and the Balla (1961) method used in the first edition, which is a subroutine in supplemental computer program B-31 noted on your diskette] have been recomputed using plane strain adjustments where $L/B > 1$. Comparing the computed q_{ult} to the measured values indicates none of the several theories/methods has a significant advantage over any other in terms of a best prediction. The use of ϕ_{ps} instead of ϕ_{tr} when $L/B > 1$ did improve the computed q_{ult} for all except the Balla method.

Since the soil wedge beneath round and square bases is much closer to a triaxial than plane strain state, the adjustment of ϕ_{tr} to ϕ_{ps} is recommended only when $L/B > 2$.

inclination factors i_i [both noted in discussion of Eq. (*j*)] for cases where the footing load is inclined from the vertical. These additions produce equations of the general form shown in Table 4-1, with select N factors computed in Table 4-4. Program BEARING is provided on disk for other N_i values.

Meyerhof obtained his N factors by making trials of the zone abd' with arc ad' of Fig. 4-3*b*, which include an approximation for shear along line cd of Fig. 4-3*a*. The shape, depth, and inclination factors in Table 4-3 are from Meyerhof (1963) and are somewhat different from his 1951 values. The shape factors do not greatly differ from those given by Terzaghi except for the addition of s_q. Observing that the shear effect along line cd of Fig. 4-3*a* was still being somewhat ignored, Meyerhof proposed depth factors d_i.

He also proposed using the inclination factors of Table 4-3 to reduce the bearing capacity when the load resultant was inclined from the vertical by the angle θ. When the i_γ factor is used, it should be self-evident that it does not apply when $\phi = 0°$, since a base slip would occur with this term—even if there is base cohesion for the i_c term. Also, the i_i factors all $=$ 1.0 if the angle $\theta = 0$.

Up to a depth of $D \approx B$ in Fig. 4-3*a*, the Meyerhof q_{ult} is not greatly different from the Terzaghi value. The difference becomes more pronounced at larger D/B ratios.

Hansen's Bearing-Capacity Method

Hansen (1970) proposed the general bearing-capacity case and N factor equations shown in Table 4-1. This equation is readily seen to be a further extension of the earlier Meyerhof (1951) work. Hansen's shape, depth, and other factors making up the general bearing-capacity equation are given in Table 4-5. These represent revisions and extensions from earlier proposals in 1957 and 1961. The extensions include base factors for situations in which the footing is tilted from the horizontal b_i and for the possibility of a slope β of the ground supporting the footing to give ground factors g_i. Table 4-4 gives selected N values for the Hansen equations together with computation aids for the more difficult shape and depth factor terms. Use program BEARING for intermediate N_i factors, because interpolation is not recommended, especially for $\phi \gtrsim 35°$.

Any of the equations given in Table 4-5 not subscripted with a V may be used as appropriate (limitations and restrictions are noted in the table). The equations shown in this table for inclination factors i_i will be considered in additional detail in Sec. 4-6.

Note that when the base is tilted, V and H are perpendicular and parallel, respectively, to the base, compared with when it is horizontal as shown in the sketch with Table 4-5.

For a footing on a slope both the Hansen and Vesić g_i factors can be used to reduce (or increase, depending on the direction of H_i) the bearing capacity using N factors as given in Table 4-4. Section 4-9 considers an alternative method for obtaining the bearing capacity of footings on a slope.

The Hansen equation implicitly allows any D/B and thus can be used for both shallow (footings) and deep (piles, drilled caissons) bases. Inspection of the $\bar{q}N_q$ term suggests a great increase in q_{ult} with great depth. To place modest limits on this, Hansen used

$$\left.\begin{aligned} d_c &= 1 + 0.4\frac{D}{B} \\ d_q &= 1 + 2\tan\phi(1 - \sin\phi)^2\frac{D}{B} \end{aligned}\right\} \frac{D}{B} \le 1$$

N_γ values shows the following:

ϕ	Terzaghi* (1943)	Bolton and Lau (1993)	Kumbhojkar (1993)	Table 4-2 (this text)
34°	36	43.5	32	36
48	780	638	650.7	780.1

*See Terzaghi (1943), Fig. 38 and page 128.

Fortunately the N_γ term does not make a significant contribution to the computed bearing capacity, so any of the values from Tables 4-2 or 4-4 can be used (or perhaps an average).

Bolton and Lau (1993) produced new N_q and N_γ values for strip and circular footings for both smooth and rough ground interfacings. Their N_q values for either smooth or rough strips are little different from the Hansen values for rough strips. The N_q values for circular footings range to more than two times the strip values. The N_γ values for rough footings compare well with the Vesić values in Table 4-4. Since the Table 4-4 values have shape s_i and depth d_i factors to be applied, it appears that these "new" values offer little advantage and are certainly more difficult to compute (see comparison with Terzaghi values in preceding table).

Meyerhof's Bearing-Capacity Equation

Meyerhof (1951, 1963) proposed a bearing-capacity equation similar to that of Terzaghi but included a shape factor s_q with the depth term N_q. He also included depth factors d_i and

TABLE 4-4

Bearing-capacity factors for the Meyerhof, Hansen, and Vesić bearing-capacity equations

Note that N_c and N_q are the same for all three methods; subscripts identify author for N_γ

ϕ	N_c	N_q	$N_{\gamma(H)}$	$N_{\gamma(M)}$	$N_{\gamma(V)}$	N_q/N_c	$2\tan\phi(1-\sin\phi)^2$
0	5.14*	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10.9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0.276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

* = $\pi + 2$ as limit when $\phi \to 0°$.

Slight differences in above table can be obtained using program BEARING.EXE on diskette depending on computer used and whether or not it has floating point.

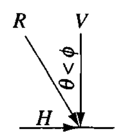
TABLE 4-2
Bearing-capacity factors for the Terzaghi equations

Values of N_γ for ϕ of 0, 34, and **48°** are original Terzaghi values and used to back-compute $K_{p\gamma}$

ϕ, deg	N_c	N_q	N_γ	$K_{p\gamma}$
0	5.7*	1.0	0.0	10.8
5	7.3	1.6	0.5	12.2
10	9.6	2.7	1.2	14.7
15	12.9	4.4	2.5	18.6
20	17.7	7.4	5.0	25.0
25	25.1	12.7	9.7	35.0
30	37.2	22.5	19.7	52.0
34	52.6	36.5	36.0	
35	57.8	41.4	42.4	82.0
40	95.7	81.3	100.4	141.0
45	172.3	173.3	297.5	298.0
48	258.3	287.9	780.1	
50	347.5	415.1	1153.2	800.0

*$N_c = 1.5\pi + 1$. [See Terzaghi (1943), p. 127.]

TABLE 4-3
Shape, depth, and inclination factors for the Meyerhof bearing-capacity equations of Table 4-1

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p\frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L}$	$\phi > 10°$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta°}{\phi°}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$

Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2
θ = angle of resultant R measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.
B, L, D = previously defined

so that the shear resistance along cd of Fig. 4-3a could be neglected. Table 4-1 lists the Terzaghi equation and the method for computing the several N_i factors and the two shape factors s_i. Table 4-2 is a short table of N factors produced from a computer program and edited for illustration and for rapid use by the reader. Terzaghi never explained very well how he obtained the $K_{p\gamma}$ used to compute the bearing-capacity factor N_γ. He did, however, give a small-scale curve of ϕ versus N_γ and three specific values of N_γ at $\phi = 0$, 34, and 48° as shown on Table 4-2. The author took additional points from this curve and used a computer to back-compute $K_{p\gamma}$ to obtain a table of best-fit values from which the tabulated values of N_γ shown in Table 4-2 could be computed from the equation for N_γ shown in Table 4-1. Inspection of Table 4-4 indicates that the Meyerhof $N_{\gamma(M)}$ values are fairly close except for angles of $\phi > 40°$. Other approximations for N_γ include the following:

$$N_\gamma = 2(N_q + 1)\tan\phi \qquad \text{Vesić (1973)}$$
$$N_\gamma = 1.1(N_q - 1)\tan 1.3\phi \qquad \text{Spangler and Handy (1982)}$$

The N_γ value has the widest suggested range of values of any of the bearing-capacity N factors. A literature search reveals

$$38 \le N_\gamma \le 192 \qquad \text{for } \phi = 40°$$

In this textbook values from Tables 4-2 and 4-4 give a range from about 79 to 109.

Recently Kumbhojkar (1993) presented a series of values of N_γ with the claim they are better representations of the Terzaghi values than those of Table 4-2. An inspection of these

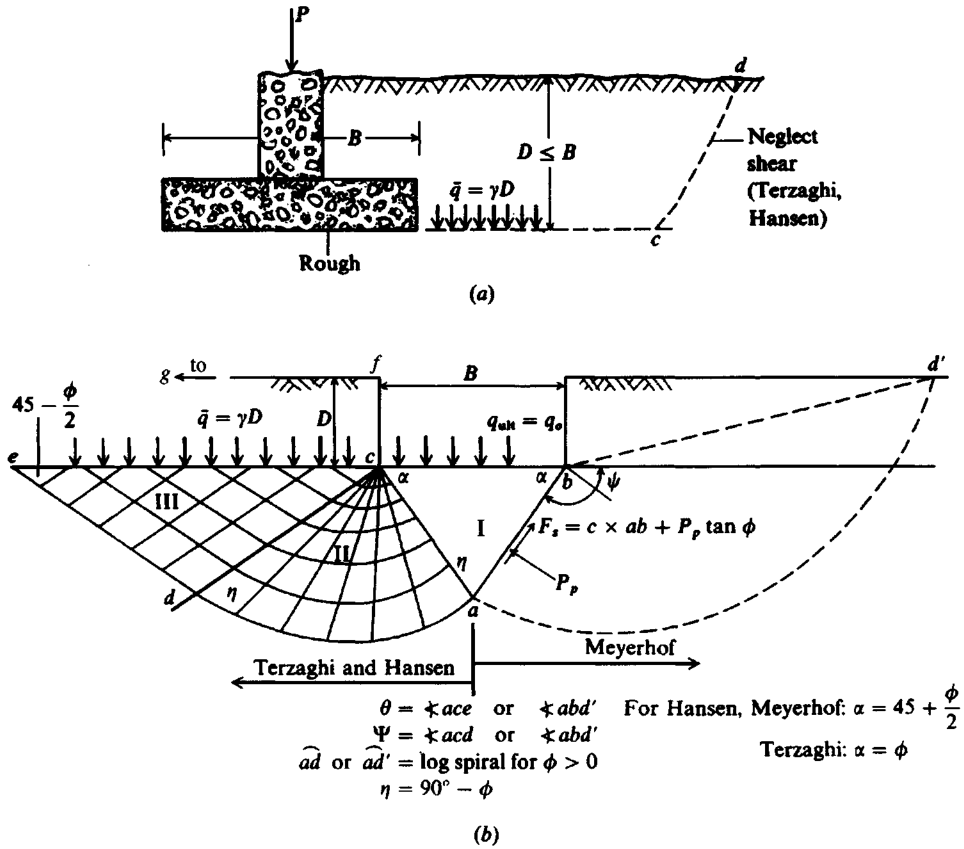


Figure 4-3 (*a*) Shallow foundation with rough base defined. Terzaghi and Hansen equations of Table 4-1 neglect shear along *cd*; (*b*) general footing-soil interaction for bearing-capacity equations for strip footing—left side for Terzaghi (1943), Hansen (1970), and right side Meyerhof (1951).

oped by Prandtl (ca. 1920) from using the theory of plasticity to analyze the punching of a rigid base into a softer (soil) material. Similar to Eq. (*k*), the basic equation was for the case in which a unit width from a long strip produced a plane strain case, all shape factors $s_i = 1.00$, but the N_i factors were computed differently. Terzaghi used $\alpha = \phi$ in Figs. 4-2 and 4-3 whereas most other theories use the $\alpha = 45 + \phi/2$ shown. We see in Table 4-1 that Terzaghi only used shape factors with the cohesion (s_c) and base (s_γ) terms. The Terzaghi bearing-capacity equation is developed, as was Eq. (*k*), by summing vertical forces on the wedge *bac* of Fig. 4-3. The difference in N factors results from the assumption of the log spiral arc *ad* and exit wedge *cde* of Fig 4-3. This makes a very substantial difference in how P_p is computed, which in turn gives the different N_i values. The shear slip lines shown on Fig. 4-3 qualitatively illustrate stress trajectories in the plastic zone beneath the footing as the ultimate bearing pressure is developed.

Terzaghi's bearing-capacity equations were intended for "shallow" foundations where

$$D \leq B$$

The Terzaghi Bearing-Capacity Equation

One of the early sets of bearing-capacity equations was proposed by Terzaghi (1943) as shown in Table 4-1. These equations are similar to Eq. (*k*) derived in the previous section, but Terzaghi used shape factors noted when the limitations of the equation were discussed. Terzaghi's equations were produced from a slightly modified bearing-capacity theory devel-

TABLE 4-1
Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for $K_{p\gamma}$ values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma B N_\gamma s_\gamma \qquad N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
$s_c =$	1.0	1.3	1.3
$s_\gamma =$	1.0	0.6	0.8

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

Vertical load: $q_{ult} = cN_c s_c d_c + \bar{q}N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined load: $q_{ult} = cN_c d_c i_c + \bar{q}N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

$$N_q = e^{\pi\tan\phi}\tan^2\left(45 + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q}N_q s_q d_q i_q g_q b_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14 s_u(1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 1.5(N_q - 1)\tan\phi$

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors.
Use Hansen's equations above.

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 2(N_q + 1)\tan\phi$

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

†See Sec. 4-6 when $i_i < 1$.